HALL AND HEAT SOURCE EFFECTS OF FLOW PAST A PARABOLIC ACCELERATED ISOTHERMAL VERTICAL PLATE IN THE PRESENCE OF CHEMICAL REACTION AND RADIATION

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Abstract

We study Hall current, heat source with radiation and chemical reaction of first order viscous fluid flow, incompressible fluid with heat and mass transfer past an accelerated isothermal vertical plate. The inverse Laplace transform technique is used to solve the ascendant mathematical statement. The numerical values are given after our study of the acceleration, thermal reading and adsorption for certain parameters, including thermal Grashof number, Prantdl number, Schmidt number, and mass Grashof number. Based on the study, we found out that the velocity of the fluid increases with increase in heat, Hall current as well as Grashof value and it decreases with increase in radiation. Concentration reduces when chemical reaction increases.

1. Introduction

The evaluation of fluid movement is an important part of the reactor heat transfer as it can be used to a wide range of systems, including biological systems, household appliances, homes and businesses, industrial operations, and food preparation like electronic equipment cooling, formation of heating and coolant systems, refrigeration of food among many others. Das et al. [1] studied how a first order homogeneous chemical reaction would alter an irregular fluid flow. Muthukumaraswamy [2] made a similar study regarding how the change in reaction affects the velocity. Sarki and Ahmed [3] also made similar study and found that the velocity of fluid increases with increase in *Gr*, *K*, *t* and *Gc*, while Thamizhsudar et al. [4] observed that axial velocity increases with increase in Hall parameter, mass as well as Grashof number. Dilip Jose and Selvaraj [5] found that the velocity increases with increase in *Gr* and *Gc*. Uwanta and Sani [6] analyzed how the parameters of the thermal *Gr*, *Gc*, *t* and the variable of thermal conductivity cause velocities to rise while the parameters of the *Pr*, *Sc*, *R*, *k* and magnetic field cause velocities to decrease. While temperature reduces with increasing Prandtl number, radiation, and suction factors, it rises with increase in thermal conductivity and heat source characteristics. With increase in *Sc* and

k, the concentration reduces. Maran et al. [7] presented graphical estimation of temperature, float speed which clearly conveys that an executed attractive region's tendency edge increases with declining speed. Rachna [8] carried a fine theoretical work obtaining the velocity to rise with increase in *Gr* and *Gc*. The impacts of the non-uniform heat parameter on dynamics are depicted in chart by Abel and Mahesha [9]. The numerical technique on several parameters of heat radiation was obtained by Ferdows et al. [10]. The Hall current effect on unsteady hydromagnetic flow was studied by Acharya et al. [11]. Siddheshwar and Mahabaleshwar [12] talked about how heat transport over a stretched sheet and MHD flow of a viscoelastic liquid is affected by radiation and heat sources. Sharma and Singh [13] described how heat-generating system is subjected to a transverse magnetic field. Muthucumaraswamy and Geetha [14] investigated the parabolic motion effects on an isothermal vertical plate. The inverse Laplace transform is solved in Hetnarski's Zastosowania Metamatikyki VII paper [15, 16].

2. Numerical Formulation

Here, we assume viscous, incompressible fluid that conducts current flowing past an infinite plate that is located in the plane $z = 0$. The *y*-axis is normal to other axes, while *x*-axis is measured in the object's drift order. This plate is parabolic accelerated along the *x*-axis with a velocity of $q = t^2$. The plate in this instance is not electrically conductive. In the flow field, the pressure is uniform as well. The continuity equation notes the elements of '*F*' velocity vector. $w' = 0$ is satisfied when $F = 0$ results in $w' = 0$ everywhere in the flow. Here, just *z* and *t* determine the flow volumes. The following equations regulate the unsteady flow under these presumptions:

$$
\frac{\partial u^*}{\partial t^*} = \vartheta \frac{\partial^2 u^*}{\partial z^{*2}} + g \beta (T^* - T^*_{\infty}) + g \beta (C^* - C^*_{\infty}) - \frac{\sigma B_0^2 \mu^2 (u^* + m_1 v^*)}{\rho (1 + m_1^2)}, \quad (1)
$$

$$
\frac{\partial v^*}{\partial t^*} = \vartheta \frac{\partial^2 u^*}{\partial z^{*2}} + \frac{\sigma B_0^2 \mu^2 (m_1 u^* - v^*)}{\rho (1 + m_1^2)},
$$
\n(2)

$$
\frac{\partial \theta^*}{\partial t^*} = \frac{1}{Pr} \frac{\partial^2 \theta^*}{\partial z^{*2}} - R\theta^* + Q\theta^*,
$$
\n(3)

$$
\frac{\partial C^*}{\partial t^*} = \frac{1}{Sc} \frac{\partial^2 C^*}{\partial z^{*2}} - kC^*.
$$
 (4)

The boundary conditions are

$$
u^* = 0, v^* = 0, T^* = T^*, C^* = C^* \text{ at } t^* \le 0, \text{ for every } z_\infty \le 0,
$$

$$
u^* = t^{*2}, T^* = T^* \text{ at } C^* = C^* \text{ at } t^* > 0 \text{ for } z^* = 0,
$$

$$
u^* \to 0, T^* \to T^* \text{ at } C^* \to C^* \text{ at } z^* \to \infty.
$$
 (5)

The consequent dimensionless aggregate is:

$$
U = \frac{u^*}{(Vu_0)^{\frac{1}{3}}}, V = \frac{v^*}{(Vu_0)^{\frac{1}{3}}}, t = t^* \left(\frac{(u_0^2)}{v}\right)^{\frac{1}{3}}, Z = z^* \left(\frac{(u_0^2)}{v^2}\right)^{\frac{1}{3}},
$$

\n
$$
\theta = \frac{T^* - T^*_{\infty}}{T^*_{w} - T^*_{\infty}}, Gr = \frac{f\beta(T^*_{w} - T^*_{\infty})}{u_0}, C = \frac{C^* - C^*_{\infty}}{C^*_{w} - C^*_{\infty}}, Gc = \frac{g\beta(C^* - C^*_{\infty})}{C^*_{w} - C^*_{\infty}},
$$

\n
$$
Pr = \frac{\mu C_p}{k}, k = K_1 \left(\frac{v}{u_0^2}\right)^{\frac{1}{3}}, Sc = \frac{v}{D}, M^2 = \frac{\sigma B_0^2}{\rho} \left(\frac{v}{u_0^2}\right)^{\frac{1}{3}}.
$$
 (6)

From the above, it is clear that we should determine the values of thermal layer transfer and proportionate heat transfer when measuring velocity since *Pr* is the ratio between momentum and thermal diffusivity. The heat transfer known as the *Grashof number* calculates the buoyancy to viscosity ratio. Since the buoyant force, as opposed to the viscous force, is mostly responsible for the convection, it is appropriate to measure the fluid to demonstrate this.

To examine the diffusion coefficient, use Schmidt as the ratio between mass diffusivity and momentum. First order chemical reaction on flow past a parabolic with rotation is shown using coupled partial differential equations. Complex velocity $q = u + iv$ was used to solve equations (1) and (2), which were then combined into one equation:

$$
\frac{\partial q}{\partial t} = Gr\theta + GcC + \frac{\partial^2 q}{\partial z^2} - mq, \text{ where } m = \frac{M^2}{(1 + h_i)},
$$

$$
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta^*}{\partial z^2} - R\theta^* + Q\theta^*,
$$

$$
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C^*}{\partial z^2} - kC
$$
 (7)

with conditions

$$
q = 0, \theta = 0, C = 0 \text{ for all } z, t \le 0,
$$

$$
q = t^2, \theta = 1, C = 1 \text{ for all } z, t = 0,
$$

$$
q \to 0, \theta \to 0, C \to 0 \text{ as } z \to \infty.
$$

(8)

3. Elucidation of the Problem

Solving equation (7) using (8) with the aid of Laplace transforms, we obtain

 $q(z)$

$$
= \sqrt{\left[\frac{\eta^2 t}{m} + t^2\right] \frac{1}{2} \left[\frac{e^{2\eta\sqrt{t}\sqrt{m}} erfc(\eta - \sqrt{mt})}{+ e^{2\eta\sqrt{t}\sqrt{m}} erfc(\eta + \sqrt{mt})}\right]} - \sqrt{\left[\frac{1}{4m} - t\right] \frac{2\eta\sqrt{t}}{2\sqrt{m}} \left[\frac{e^{-2\eta\sqrt{t}\sqrt{m}} erfc(\eta - \sqrt{mt})}{- e^{2\eta\sqrt{t}\sqrt{m}} erfc(\eta + \sqrt{mt})}\right]} - \frac{2}{m} \sqrt{\frac{t}{\pi}} e^{\left(\frac{-z^2}{4t} - mt\right)}
$$

$$
-\frac{Gr}{a(1-Pr)}\begin{bmatrix} \frac{e^{at}}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{a+m}} erfc(\eta - \sqrt{(a+m)t}) \\ +e^{2\eta\sqrt{t}\sqrt{a+m}} erfc(\eta + \sqrt{(a+m)t}) \end{bmatrix} \\ -\frac{1}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{m}} erfc(\eta - \sqrt{m t}) \\ +e^{2\eta\sqrt{t}\sqrt{m}} erfc(\eta + \sqrt{m t}) \end{bmatrix} \end{bmatrix}
$$

$$
-\frac{Gs}{b(1-Sc)}\begin{bmatrix} \frac{e^{at}}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{b+m}} erfc(\eta - \sqrt{(b+m)t}) \\ +e^{2\eta\sqrt{t}\sqrt{b+m}} erfc(\eta + \sqrt{(b+m)t}) \end{bmatrix} \\ -\frac{1}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{m}} erfc(\eta + \sqrt{(b+m)t}) \\ +e^{2\eta\sqrt{t}\sqrt{m}} erfc(\eta + \sqrt{m t}) \end{bmatrix} \end{bmatrix}
$$

$$
+\frac{Gr}{a(1-Pr)}\begin{bmatrix} \frac{e^{at}}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{Pr(a+R-Q)}} erfc(\eta\sqrt{Pr} - \sqrt{(a+R-Q)t}) \\ +e^{2\eta\sqrt{t}\sqrt{Pr(a+R-Q)}} erfc(\eta\sqrt{Pr} + \sqrt{(a+R-Q)t}) \end{bmatrix} \\ -\frac{1}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{Pr(R-Q)}} erfc(\eta\sqrt{Pr} - \sqrt{(R-Q)t}) \\ +e^{2\eta\sqrt{t}\sqrt{Pr(R-Q)}} erfc(\eta\sqrt{Pr} + \sqrt{(R-Q)t}) \end{bmatrix} \end{bmatrix}
$$

$$
+\frac{Gc}{b(1-Sc)}\begin{bmatrix} \frac{e^{bt}}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{Sc(b+k)}} erfc(\eta\sqrt{Sc} - \sqrt{(b+k)t}) \\ +e^{2\eta\sqrt{t}\sqrt{Sc(b+k)}} erfc(\eta\sqrt{Sc} - \sqrt{(b+k)t}) \end{bmatrix} \\ -\frac{1}{2} \begin{bmatrix} e^{-2\eta\sqrt{t}\sqrt{Sc(b+k)}} erfc(\eta\sqrt{Sc} - \sqrt{k t}) \\ +e^{2\eta\sqrt{t}\sqrt{Sc}} er
$$

$$
+ e^{2\eta\sqrt{Pr}\sqrt{(R-Q)t}} erfc(\eta\sqrt{Pr} + \sqrt{(R-Q)t})],
$$
\n(10)

$$
C = \frac{1}{2} \left[e^{-2\sqrt{m}\sqrt{Sck}} erfc(\eta\sqrt{Sc} - \sqrt{kt}) + e^{2\sqrt{m}\sqrt{Sck}} erfc(\eta\sqrt{Sc} + \sqrt{kt}) \right].
$$
 (11)

4. Results and Discussion

The velocity for changing values of *k*, *Sc*, *Pr*, *Gr*, *Gc*, *h*, *R* and *Q* has been presented in the diagrams given in this section.

Figure 1. Concentration profile for many *k* values, including 5, 8, and 11. As chemical reaction *k* grows, concentration decreases.

Figure 2. It shows the *Sc* concentration profile. Concentration decreases as the Schmidt number increases, for various values of $Sc = 0.3, 0.6$ and 2.01.

Figures 1 and 2 show the concentration profiles.

Figure 3. Temperature profile for $Pr = 0.71, 7.0$. Temperature shrinks when Prandtl number *Pr* hikes.

Figure 4. Temperature profile for $R = 0.5, 2.5, 4.0, 5.5, 9.5, 17$. Temperature shrinks when radiation *R* rises.

Figure 5. Temperature profile for $Q = 1, 4, 5, 7, 8, 11$. Temperature rises when heat source *Q* rises.

In Figures 3, 4 and 5, the temperature profiles are depicted.

Figure 6. PV for $Gc = 5, 7, 10$. The velocity rises when mass Grashof number *Gc* hikes.

Figure 7. SV for $Gc = 11, 14, 17$. The velocity rises when mass Grashof number *Gc* hikes.

Figure 8. PV for $Gr = 5, 7, 10$. The velocity increases when thermal Grashof values rise.

Figure 9. SV for $Gr = 11, 14, 17$. The velocity rises when thermal Grashof values rise.

Figure 10. PV for $h = 1, 1.5, 2$. The velocity rises when heat source rises.

Figure 11. SV for $h = 1.75, 2.5, 4$. The velocity rises when heat source rises.

Figure 12. PV for $k = 1, 3, 5$. The velocity reduces when *k* rises.

Figure 13. SV for $k = 8, 10, 12$. The velocity reduces when *k* rises.

Figure 14. PV for $Q = 1, 4, 7$. The velocity reduces when heat source rises.

Figure 15. SV for $Q = 5$, 8, 11. The velocity reduces when heat source rises.

Figure 16. PV for $M = 5, 6, 7$. The velocity reduces with increasing Hartmann number.

Figure 17. SV for $M = 8, 10, 12$. The velocity decreases with increasing Hartmann number.

Figure 18. PV for different values of *R*. The velocity reduces when *R* rises.

Figure 19. SV for $R = 5.5$, 9.5, 17. The velocity reduces when *R* rises.

Figure 20. Velocity for $Pr = 0.71, 7.0$. The velocity reduces when Prantdl number *Pr* rises.

Figure 21. Velocity for $Sc = 0.3, 0.6, 2.01$. The velocity reduces when Schmidt (*Sc*) number rises.

In Figures 6 to 21, the velocity profiles are depicted.

5. Tabulation

Non-dimensional parameter	Figure 2 Figure 1		
	Concentration k	Concentration Sc	
Gr	7	7	
Gc	7	7	
Pr	0.71	0.71	
\boldsymbol{R}	5	5	
Q	1	1	
\boldsymbol{M}	5.5	5.5	
\boldsymbol{h}	2	$\overline{2}$	
Sc	2.01	0.3, 0.6, 2.01	
\boldsymbol{k}	5, 8, 11		
1			

Table 1. The estimated concentration profile

Table 2. Numerical estimated temperature profiles for several values of *Pr*, *R*, and *Q*

Table 3. Numerical estimated concentration profiles for several values of *Gc* and *Gr*

Non-dimensional parameters	Figure 6	Figure 7	Figure 8	Figure 9
	Primary	Secondary	Primary	Secondary
	velocity Gc	velocity Gc	velocity Gr	velocity Gr
Gr	7	7	5, 7, 10	11, 14, 17
Gc	5, 7, 10	11, 14, 17	7	7
Pr	0.71	0.71	0.71	0.71
\boldsymbol{R}	5	5	5	5
Q	1	1	1	1
\boldsymbol{M}	5.5	5.5	5.5	5.5
\boldsymbol{h}	$\overline{2}$	$\overline{2}$	$\overline{2}$	$\overline{2}$
Sc	2.01	2.01	2.01	2.01
\boldsymbol{k}	1	1	1	1
t	1	1		

Non-dimensional parameters	Figure 10	Figure 11	Figure 12	Figure 13
	Primary	Secondary	Primary	Secondary
	velocity h	velocity h	velocity k	velocity k
Gr	7	7	7	7
Gc	7	7	7	7
Pr	0.71	0.71	0.71	0.71
\boldsymbol{R}	5	5	5	5
ϱ	1	1	1	1
\boldsymbol{M}	5.5	5.5	5.5	5.5
\boldsymbol{h}	1, 1.5, 2	1.75, 2, 4	2	$\overline{2}$
$\mathcal{S}c$	2.01	2.01	2.01	2.01
k	1	1	1, 3, 5	8, 10, 12
t	0.2	0.2	0.2	0.2

Table 4. Numerical estimated concentration profiles for several values of *h* and *k*

Table 6. Numerical values for estimated velocity profiles for varying *M*, *Sc* and *Pr* values

6. Conclusion

As this is a variational study from the literature involving accelerated isothermal vertical plate with the basic HMT aspects, this provides a simple and nice platform for computational work and based on the calculations, we could conclude that

(i) Velocity reduces when radiation '*R*' rises,

Velocity reduces when Hartmann number *M* rises and

Velocity rises when Grashof '*Gc*' and '*Gr*' values rise,

Velocity rises when heat source '*Q*' rises,

Velocity rises when Hall current '*h*' rises.

(ii) Temperature falls when radiation '*R*' rises and

Temperature rises when heat source '*Q*' rises.

(iii) Concentration reduces when chemical reaction '*k*' increases.

So, we are able to achieve an extended list of conclusions by the variation and we intend to enhance the study by including more parameters in our future study.

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